Passive Localization Method of LFM Signal Transmitter based on Multi-channel Joint Accumulation in FrFT Domain

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***Abstract*—The traditional two-step localization method for transmitter requires to estimate the signal parameters such as angle of arrival (AOA) and time of arrival (TOA), wich confronts the problem of localization error cumulation. While the direct position determination (DPD) can effectively reduce the estimation error and achieve superior localization performance than the two-step localization, which is widely applied in passive radar. Unfortunately, the uncertainty of the transmission signal parameters will causes the localization performance degradation for DPD method in passive localization. To address these issues, this paper considers the LFM signal transmitter localization with passive radar. Firstly, the signal within each receiving channel is accumulated by fractional Fourier transform (FrFT). Then the signal envelope alignment of each channel is performed in the FrFT domain by using the characteristics of FrFT, so as to realize the multi-channel signal accumulation. Finally, the transmitter is accurately localized by the two-dimensional position search. Simulation results show that the proposed method outperforms the two-step method and DPD method in low SNR.**

***Index Terms*—Passive localization, LFM signal transmitter, Fractional Fourier transform, Multichannel signal accumulation**

I. Introduction

The passive radar system has the characteristics of high concealment and low interception because it does not have radiation signal itself [2]. Passive radar needs to analyze signals from external radiation sources, and linear frequency modulation (LFM) signals are widely used in various radiation sources because of its large time-bandwidth product and has anti-interference ability [3]. Therefore, it is important to accurately locate the LFM signal transmitter. Transmitter location methods in passive radar systems are generally divided into two categories, including two-step localization algorithms and direct localization.

The first process of the two-step localization algorithm is to measure the covariates related to the target position from the receive signal, including the phase difference [4] [5], the time difference of arrival (TDOA) [13], frequency different of arrival (FDOA) [7], doppler rate [8], direction of arrival (DOA) [9] and received signal strength (RSS) [10]. The second process of two-step localization method uses these covariates to construct and solve equation set to determine target location. Solving equation set can be accomplished by exhaustive search method [11], the least squares method [12] and the pseudo-linear method [13] etc. Two-step localization algorithm has low computational complexity. However, when the signal to noise ratio (SNR) is low or the receiving nodes in the system are interfered, the localization accuracy will deteriorate drastically.

Direct position determination (DPD) algorithms use the transmitter location information embedded in the signal to construct an objective function (cost function), which obtains localization result by processing the receive signals directly. The objective function is solved by optimization algorithms such as exhaustive search [14] [15], which has higher positioning accuracy [14] in the case of low SNR. However, for non-cooperative signals, the DPD localization ignores the transmitted signal parameters, which degrades the localization performance [15].

Fractional Fourier transform (FRFT) is a generalization of the Fourier transform (FT) [16], which can be used to estimate the parameters of a signal due to its good energy concentration on LFM signals. Chen [17] uses FrFT combined with the DPD algorithm to realize LFM signal transmitter localization, which improves the localization performance and enables the estimation of the number of transmitting sources. However, the method is unable to apply the accumulation gain between channels when estimating the number of radiation sources.

In this paper, we study algorithms for localizing non-cooperative LFM signal transmitters. The received signal is processed by FrFT and the time delay characteristics for LFM pulses in FrFT domain are analyzed, followed by envelope alignment and signal accumulation in the FrFT domain. Finally, the localization is achieved by performing a two-dimensional grid search on the location.

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II. Signal Modeling and Problem description

Assume that there are M receiver stations are located at $\mathbf{P}_j = [x_j, y_j], j = 1, ..., M$, the position of LFM signal transmitter is $\mathbf{P}_i = [x_i, y_i]$. The signals received by each receiving station are sent to the processing terminal for unified processing, as shown in Fig. 1.

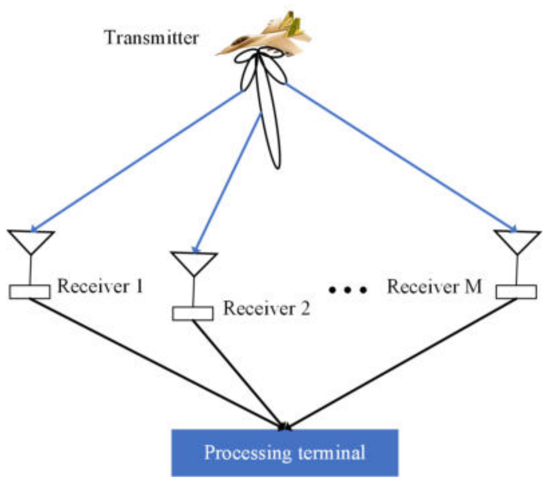


Fig. 1: Sketch map of passive localization scenario.

For the j-th receiving station, the received signal can be expressed as:

$$s_{rj}(t) = s_i(t - \tau_{ij}) + n(t), \tag{1}$$

where the transmitting signal is modeled as,

$$s_i(t; f_{0i}, k_i, T_{pi}) = A_i rect(\frac{t}{T_{pi}}) \exp\{j\pi(2f_{0i}t + k_i t^2)\}, \tag{2}$$

f_{0i}, k_i, and T_{pi} represent the initial frequency, chirp rate, and pulse width of the transmitted signal, respectively. And $n(t)$ is gaussian white noise. τ_{ij} represents the delay from the i-th transmitter to the j-th receiver:

$$\tau_{ij} = \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{c}, \tag{3}$$

with the speed of light c. The time difference of arrival between the each receiver and the reference receiver is given as:

$$\tau_{j-ref} = \frac{|\mathbf{P}_i - \mathbf{P}_{rj}|}{c} - \frac{|\mathbf{P}_i - \mathbf{P}_{ref}|}{c} = \tau_j - \tau_{ref}, \tag{4}$$

where $|\bullet|$ stands for the Euclidean distance, $\mathbf{P}_{ref}$ is the position vector of reference receiver, which can be set arbitrarily. This paper considers the case of positioning a single transmitter.

III. Algorithm description

A. FrFT characteristic for LFM pulse

FrFT has good energy concentration for LFM signals and has many applications in LFM analysis and parameter estimation. The common FRFT is defined as [16]:

$$f_p(u) = \int_{-\infty}^{+\infty} K_p(u,t) s(t) dt, \tag{5}$$

where $K_p(u,t)$ is the kernel function, we have $\alpha = \frac{p\pi}{2}$ and:

$$K_p(u,t) = \begin{cases} A_\alpha \exp\left[j\pi\left(u^2 \cot\alpha - 2ut\csc\alpha\right)\right] \\ \times \exp\left[j\pi\left(t^2 \cot\alpha\right)\right], \ \alpha \neq n\pi \\ \delta(u-t), \ \alpha \neq 2n\pi \\ \delta(u+t), \ \alpha \neq (2n+1)\pi \end{cases}. \tag{6}$$

For a signal with a duty cycle of 100%, the initial frequency f_0 and the chirp rate k of the LFM signal can be directly estimated by the following formula:

$$\begin{cases} \hat{k} = -\cot(\alpha_0) \\ \hat{f}_0 = \mu_0 \csc(\alpha_0) \end{cases}. \tag{7}$$

where $[\mu_0, a_0]$ is the position of the peak of the signal after the FrFT.

Since the integration kernel in FrFT does not contain the pulse width and time of arrival terms, for LFM pulse signals with less than 100% duty cycle, the estimated initial frequency $\hat{f}_0$ is coupled to the time of arrival and chirp rate k, as shown in Fig.2.

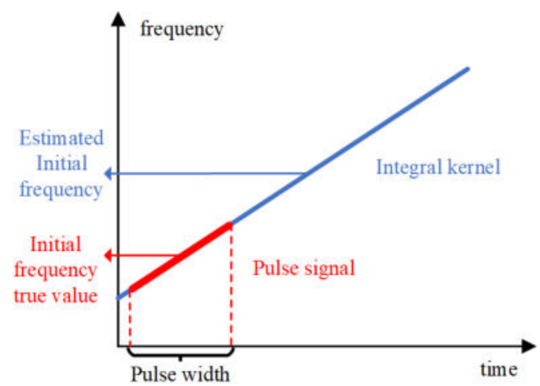


Fig. 2: The relationship between the estimated initial frequency and the true value.

The derivation of the equation of the signal initial frequency f_0 estimate with respect to each parameter is given next. Consider receiving a single LFM pulse signal $s(t)$ with the time delay τ,the FrFT results of $s(t)$ is as follows.

$$\begin{aligned} f_p(u) = & \int_{-\infty}^{+\infty} s(t-\tau) A_\alpha \\ & \times \exp\left[j\pi\left(u^2\cot\alpha - 2ut\csc\alpha + t^2\cot\alpha\right)\right] dt \\ = & \int_{-\infty}^{+\infty} A rect(\frac{t-\tau}{T_p}) \exp\{j\pi(2f_0(t-\tau) + k(t-\tau)^2)\} \\ & \times \exp\left[j\pi\left(u^2\cot\alpha - 2ut\csc\alpha + t^2\cot\alpha\right)\right] dt. \end{aligned} \tag{8}$$

When the transformation order matches the chirp rate of the signal $k = -\cot\alpha$, we have (9). According to the (9), when $(f_0 - k\tau)t = ut\csc\alpha$, $f_p(u)$ gets its maximum value. Thus, The initial frequency of the signal obtained by the estimation of (7) is,

$$\bar{f}_{es} = u\csc(\alpha) = f_0 - k\tau. \tag{10}$$

$$
\begin{aligned}
f_p(u) &= \int_{-\infty}^{+\infty} s(t-\tau) A_\alpha \exp\left[j\pi\left(u^2\cot\alpha - 2ut\csc\alpha + t^2\cot\alpha\right)\right] dt \\
&= \int_{-\infty}^{+\infty} Arect(\frac{t-\tau}{T_p}) \exp\{j\pi(2f_0(t-\tau) + k(t-\tau)^2)\} \exp\left[j\pi\left(u^2\cot\alpha - 2ut\csc\alpha + t^2\cot\alpha\right)\right] dt \\
&= \int_{-\infty}^{+\infty} Arect(\frac{t-\tau}{T_p}) \exp\{j\pi(2f_0(t-\tau) - 2kt\tau + \tau^2)\} \exp\left[j\pi\left(u^2 k - 2ut\csc\alpha\right)\right] dt \\
&= \int_{-\infty}^{+\infty} Arect(\frac{t-\tau}{T_p}) \exp[j\pi((2f_0 - 2k\tau)t - 2f_0\tau + u^2 k + \tau^2)] \times \exp\left[-j\pi 2ut\csc\alpha\right] dt \\
&= \int_{-\infty}^{+\infty} \tilde{A}rect(\frac{t-\tau}{T_p}) \exp[j2\pi(f_0 - k\tau)t] \exp\left[-j2\pi ut\csc\alpha\right] dt
\end{aligned} \tag{9}
$$

The estimate of the initial frequency is a coupling term that contains the initial frequency, the time delay, and chirp rate signal of the original signal. Thus the location of the peak point in the FrFT domain contains the time delay information. Since f_0 in (10) is unknown, thus the value of τ cannot be obtained. However, since k can be estimated by (7), it is possible to calculate the τ_{j-ref}, which is the basis of the TDOA localization algorithm.

However, the above algorithms have high requirements on the signal-to-noise ratio of each channel and large localization errors. In this paper, accurate localization of transmitter is achieved by energy accumulation of the signal in the FrFT domain.

B. Signal Accumulation in FrFT Domain and Transmitter localization

In this paper, suppose all recieving stations are perfectly synchronised, delay correction is performed by shifting in the FrFT domain in combination with (10). Receiver 1 is used as the reference receiver. Then, the difference of arrival time between transmitting signals to each receiver and to the reference receiver can be calculated. According to the tdoa of different radiation sources, the envelope alignment of the FrFT domain is carried out and the signal energy accumulation is completed, the schematic diagram of signal accumulation in FrFT domain is shown in Fig. 3.

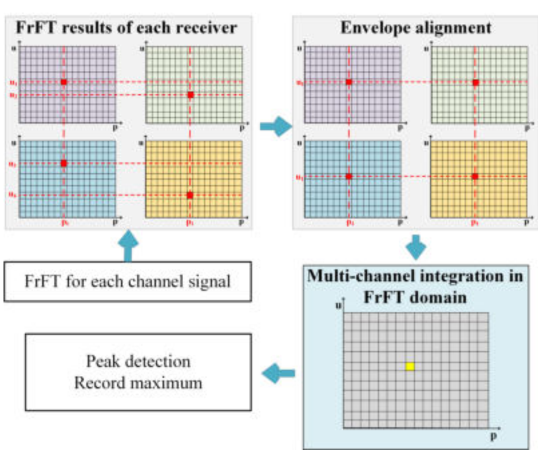


Fig. 3: Flowchart of signal accumulation in FrFT Domain.

For position $\mathbf{P_e} = [x_e, y_e]$, the TDOA between the transmitting signal of transmitter at $\mathbf{P_e}$ to the $i-th$ receiver and the reference receiver τ_{j-ref} can be caculated by (4). Considering the case of a single transmitter, the receive signal of the $j-th$ receiver is $s_{rj}(t)$, and the FrFT result of $s_{rj}(t)$ is,

$$
Fr_j(\alpha, u) = \int_{-\infty}^{+\infty} K_\alpha(u,t) s_{rj}(t) dt. \tag{11}
$$

For each transformation order α, , if α corresponds to the optimal FrFT transformation order of the signal $s_{rj}(t)$, the position difference of peak envelope between $s_{rj}(t)$ and $s_{r1}(t)$ can be calculated using (10),

$$
\begin{aligned}
u_1 - u_j &= (\bar{f}_{es1} - \bar{f}_{esj})/csc(\alpha) \\
&= (f_0 - k\tau_1 - f_0 - k\tau_j)/csc(\alpha), \\
&= -k(\tau_{j-ref})/csc(\alpha)
\end{aligned} \tag{12}
$$

where k can be obtained bt $k = -cot(\alpha)$. Therefore, for position $\mathbf{P_e}$ as well as the $j-th$ receiver, the FrFT domain peak envelope alignment can be realized by the following equation.

$$
Frs_j(\alpha, u) = Fr_j(\alpha, u + \frac{\cot\alpha}{\csc\alpha}\tau_{j-ref}). \tag{13}
$$

After aligning the envelopes of the peak points of each channel, the channels were subjected to energy accumulation and the maximum value is obtained.

$$
\ell([x_e, y_e]) = max(\sum_{j=1}^{M} |Frs_j(\alpha, u)|). \tag{14}
$$

Considering that the distance error has great influence on the phase, it is difficult to compensate the peak point of each channel, so the non-coherent accumulation method is adopted in this paper. Thus, the positioning problem is transformed into the peak detection problem of the plane represented by (14). The flow chart of the positioning algorithm proposed in this paper is shown in Fig. 4.

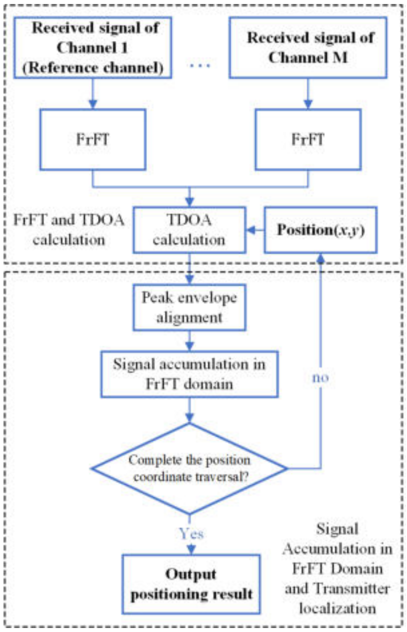


Fig. 4: Flowchart of algorithm.

The parameters of the signal can be obtained by the peak position of the accumulated FrFT domain, but this does not affect the subsequent positioning. The simulation experiment examples in this paper are only for positioning.

IV. Simulation Results

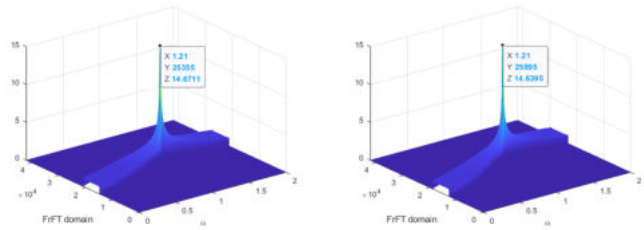
Detailed simulation results are given in this section to validate the algorithm's performance for radiated source localization of non-cooperative signals. Also, the DPD as well as Hyperbolic positioning (HP) algorithm, which is a classic two-step positioning method, are compared in order to validate the superiority of the approach. The sampling frequency of the simulation receiver and the digital down-conversion (DDC) reference frequency are 60MHz and 200MHz respectively.

Considering the single-target case, where the position coordinates of the four receivers are (-50, 20) km, (-10, 0) km, (10, 0) km, (50, 20) km, in a 2-dimensional Cartesian coordinate plane. The location of the target transmitter is (12, 60) km. The initial frequency, bandwidth (BW) and pulse width (PW) of the signal emitted by transmitter are 210 MHz, 3 MHz and 100 us respectively.

A. Noiseless scenario

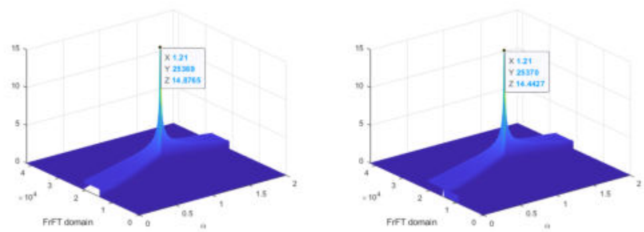
For noiseless scenario, FrFT was performed on each channel signals, and the results of 2 channel are shown in the Fig.5. And the result after the envelope alignment process is shown in Fig.6.

Two-dimensional search is performed in the detection region, and signal accumulation in FrFT domain is carried out. The results obtained according to (13) are shown in Fig.7.



(a) (b)

Fig. 5: FrFT result of receive signal in each channel (a) channel 1. (b) channel 2.



(a) (b)

Fig. 6: Result of envelope alignment in FrFT domain (a) channel 1. (b) channel 2.

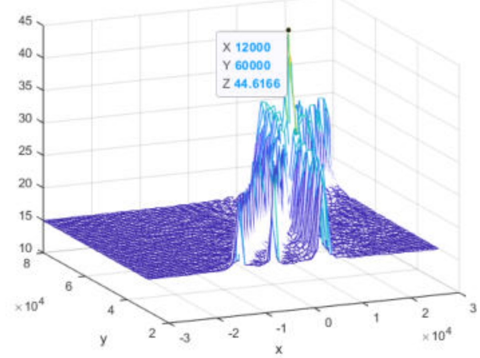
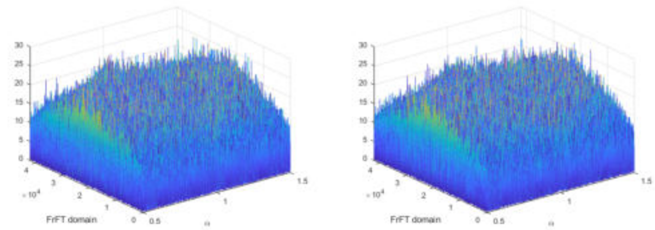


Fig. 7: Two-dimensional grid search results.

B. Low SNR scenario

The SNR of receive signal is -7dB. Fig. 8 shows the results of individual channels after FrFT, and it can be seen that at low signal-to-noise ratios, the peak energy accumulation can no longer be obtained for a single channel.Fig. 9 shows the results after accumulating each channel and making two-dimensional grid search. It can be seen that the algorithm proposed in this paper is able to achieve good results even in the case of low SNR, while HP method and DPD method can not achieve accurate localization within low SNR of signal channel.



(a) (b)

Fig. 8: FrFT result of receive signal in low SNR (a) channel 1. (b) channel 2.

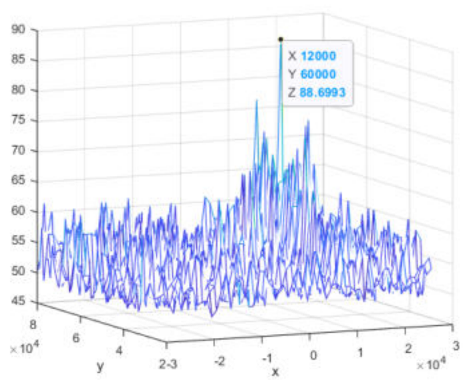


Fig. 9: Two-dimensional grid search results in low SNR.

C. Localization performance

Monte Carlo experiments were used to analyze the localization performance and to compare the DPD as well as HP methods.We focus on the position root mean square error (RMSE) defined by:

$$RMSE = \sqrt{\frac{1}{M}\sum_{i=1}^{M} |\mathbf{p} - \hat{\mathbf{p}}_i|}^2 \tag{15}$$

where $\hat{\mathbf{p}}$ represents the estimated location of the transmitter, and $\mathbf{P}$ is the true location of the transmitter, M is the number of Monte Carlo experiments. In addition, in order to analyze the effect of the number of receivers on the performance of parameter estimation, Monte Carlo simulations are performed for the cases of 6 receivers as well as 4 receivers, respectively.The simulation results are shown in Fig. 10 and Fig. 11.

It can be clearly seen that all three algorithms can perform well when the signal-to-noise ratio exceeds about -2 dB. The algorithm proposed in this paper has better localization accuracy than the other two algorithms when it is not interfered by other transmitters. As the SNR decreases, the performance of the DPD algorithm and the HP algorithm decreases much faster than that of the FrFT-Accumulation-DPD algorithm. In addition, with the reduction of the number of receivers, the positioning performance of various algorithms has a certain degree of decline, and the performance degradation of DPD and HP algorithm is more obvious, wich shows that the proposed method has good robustness.

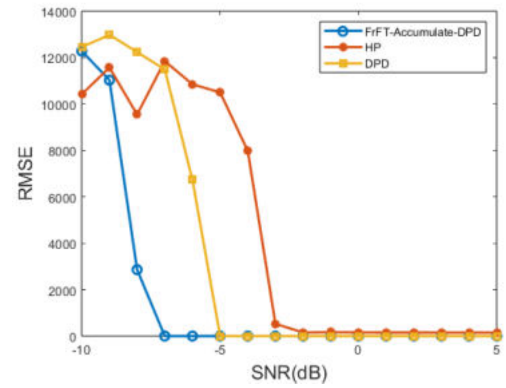


Fig. 10: The localization performance comparison for 6 receivers.

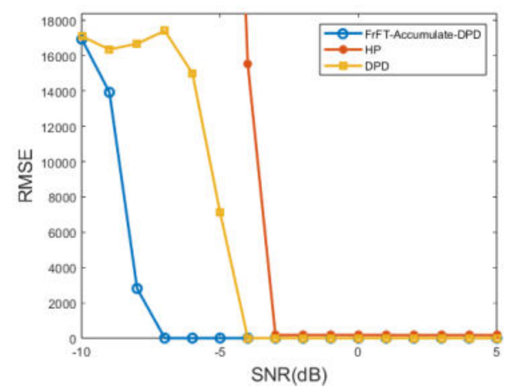


Fig. 11: The localization performance comparison for 4 receivers.

V. Conclusion

In this paper, we consider the passive localization of the LFM signal transmitter when the parameters are unknown. The FrFT-Accumulate-DPD method is proposed, which combines the properties of LFM pulse in FrFT domain and signal correlation between channels. First, the received signal of each channel is transformed into the FrFT domain, and the envelope alignment of each channel signal is carried out, then the multi-channel signal accumulation is realized. Finally, the transmitter localization is realized by two-dimensional grid search. Compared with DPD and HP, FrFT-Accumulate-DPD has better performance under the condition of low SNR for single transmitter localization.

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